Sharing is Caring

Combination of Theories

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8th International Workshop on Satisfiability Modulo Theories
July 15th, 2010, Edinburgh

Outline

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The Theory Combination Problem

Combination of Theories

Given individual decision procedures for (quantifier free) first-order theories T_1 and T_2 how can we combine them in a modular fashion into a decision procedure for a theory $T_1 \oplus T_2$?

- Nelson-Oppen Method (1978)
- Allows one to decide the combination using decision procedures for T_1 and T_2 as black-boxes
- Most SMT Solvers that involve more than one theory use a combination method based on Nelson-Oppen

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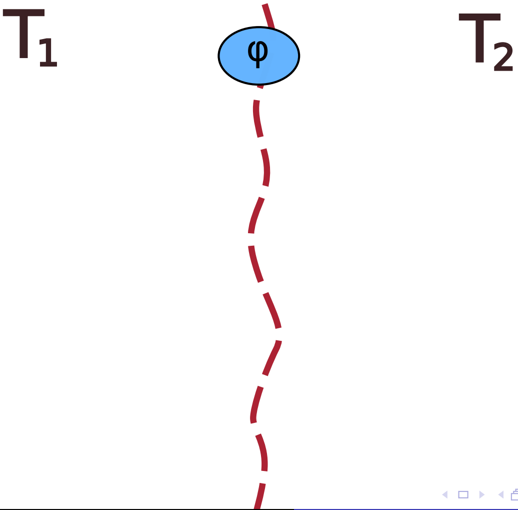
Nelson-Oppen: Idea (1978)



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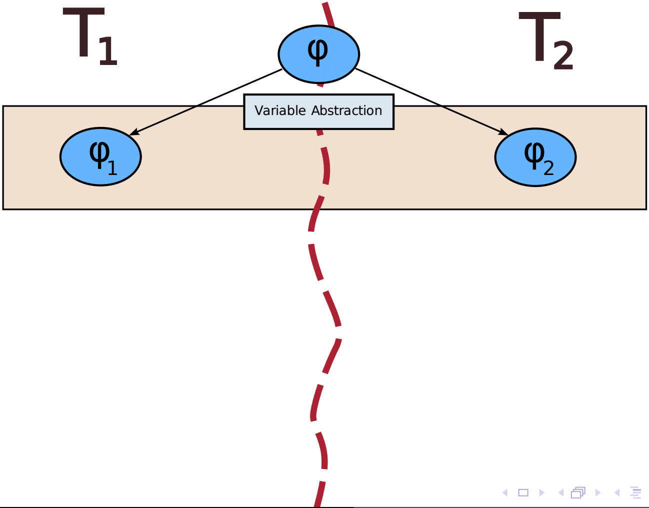

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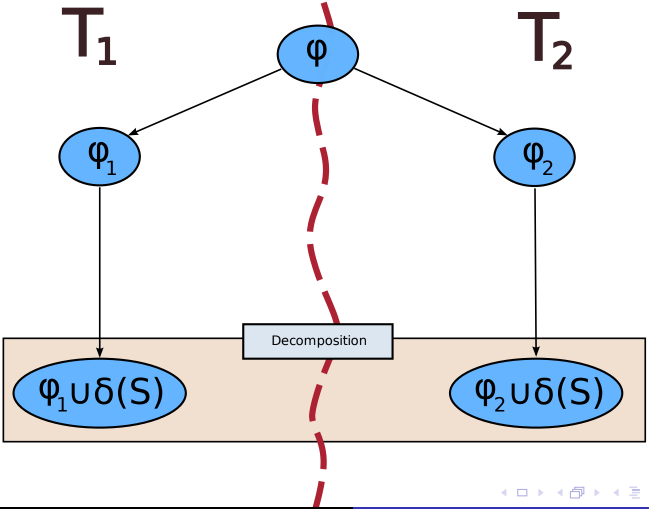

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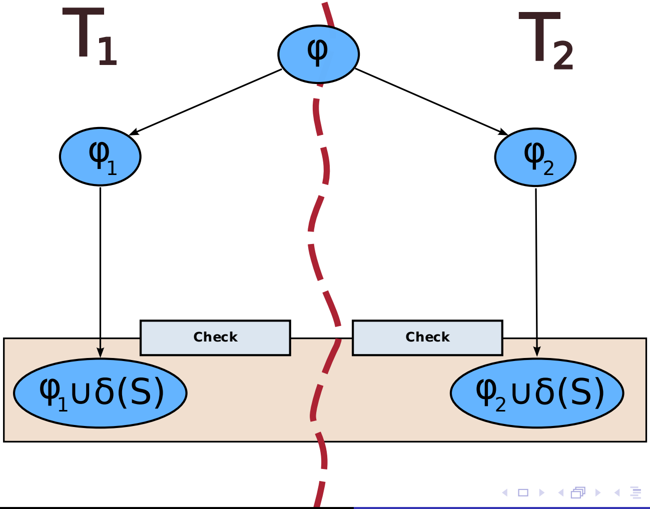

Nelson-Oppen: Example

$$\bigwedge_{i=1}^{n} (x_i = f_i(x_{i+1}) \wedge x_i = y_i \times z_i)$$

$$\phi_1 : \bigwedge_{i=1}^{n} (x_i = f_i(x_{i+1})) \qquad \phi_2 : \bigwedge_{i=1}^{n} (x_i = y_i \times z_i)$$

$$S = \{x_1, x_2, \ldots, x_n\}$$

$$\phi_1 \wedge \delta(S) \qquad \phi_2 \wedge \delta(S)$$

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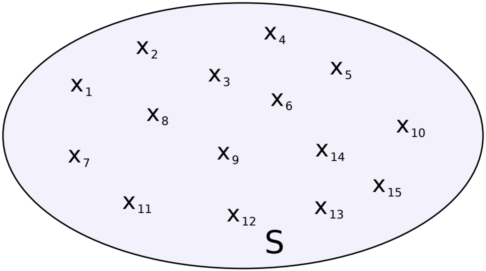
Nelson-Oppen: Complexity

Complexity

Search for a suitable arrangement over the shared variables introduces a heavy layer of complexity:

$$O(\mathcal{T}_1(n)) \oplus O(\mathcal{T}_2(n)) \implies O(2^{n^2} \times (\mathcal{T}_1(n) + \mathcal{T}_2(n))) \ .$$



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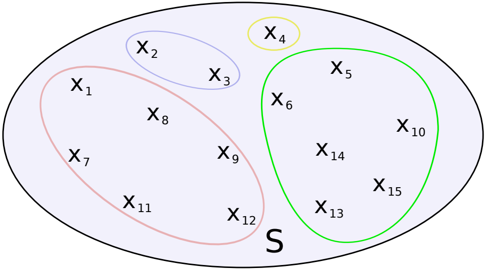

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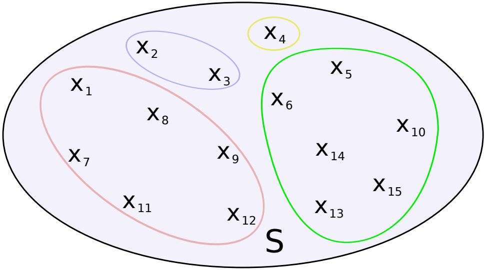

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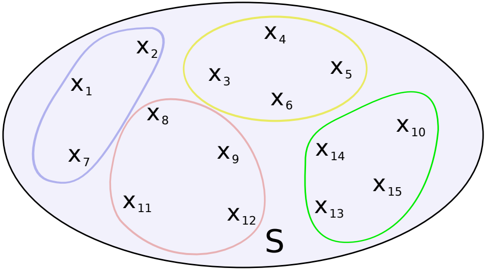

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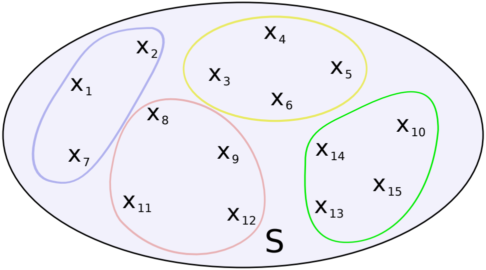

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Nelson-Oppen: Example

But...

We are free to interpret f_i as we wish so there is no need to guess an arrangement.

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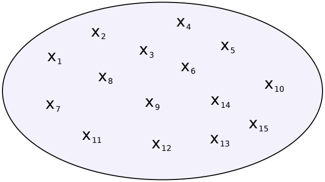
Arrangements and Care Graphs

- Instead of guessing an equivalence relation over all the shared variables we will pick the pairs of variables that we (decision procedure) are interested in (care graph).



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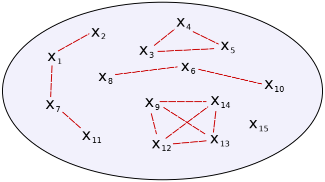

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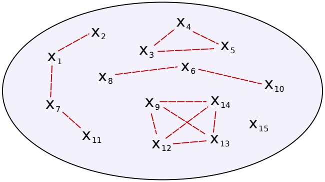

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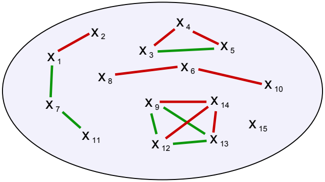

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Equality Propagation

Definition (Equality Propagator $\mathfrak{P}^{=}_{T}[\![\cdot]\!]$ for a theory T)

For every set V of variables, maps a set of T-literals ϕ into a set of equalities and dis-equalities between variables in V:

$$\begin{aligned}\mathcal{E} = \mathfrak{P}^{=}_{T}[\![V]\!](\phi) = \; & \{x_1 = y_1, x_2 = y_2, \ldots, x_m = y_m\} \; \cup \\ & \{z_1 \neq w_1, z_2 \neq w_2, \ldots, z_n \neq w_n\} \;,\end{aligned}$$

where $vars(\mathcal{E}) \subseteq V$ and

1. $\phi \implies \mathcal{E}$ is valid in T, and
2. $\mathfrak{P}^{=}_{T}[\![V]\!]$ is monotone, i.e. $\mathfrak{P}^{=}_{T}[\![V]\!](\phi) \subseteq \mathfrak{P}^{=}_{T}[\![V]\!](\phi \cup \psi)$.

Equality Propagation

Given two theory propagators:

1. $\mathfrak{P}^{=}_{T_1}[\![\cdot]\!]$ for theory T_1
2. $\mathfrak{P}^{=}_{T_1}[\![\cdot]\!]$ for theory T_2

we can construct a theory propagator $\mathfrak{P}^{=}_{T}[\![\cdot]\!]$ for the combined theory $T = T_1 \oplus T_2$

$$\mathfrak{P}^{=}_{T}[\![V]\!](\phi) = (\mathfrak{P}^{=}_{T_1}[\![V]\!] \oplus \mathfrak{P}^{=}_{T_2}[\![V]\!])(\phi) = \psi_1^* \cup \psi_2^* \ ,$$

where $\langle \psi_1^*, \psi_2^* \rangle$ is the least fixpoint of the following operator

$$\mathfrak{P}^{=}_{T}[\![V]\!]\langle \psi_1, \psi_2 \rangle = \left\langle \mathfrak{P}^{=}_{T_1}[\![V]\!](\phi_1 \cup \psi_2), \mathfrak{P}^{=}_{T_2}[\![V]\!](\phi_2 \cup \psi_1) \right\rangle \ .$$

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$$\mathfrak{P}^{=}_{T}[\![V]\!]\langle \psi_1, \psi_2 \rangle = \langle \quad {}_1)\rangle \quad .$$

Propagation

$\Psi_1^1 : P_1(\varphi_1)$ $\Psi_2^1 : P_2(\varphi_2)$

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Propagation

$\Psi_1^1 : P_1(\varphi_1)$ $\qquad$ $\Psi_2^1 : P_2(\varphi_2)$

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$\psi_1^3 : P_1(\varphi_1 \cup \psi_2^2)$ $\qquad$ $\psi_2^3 : P_2(\varphi_2 \cup \psi_2^2)$

$\vdots$

$\boxed{\psi_1^n : P_1(\varphi_1 \cup \psi_1^n) \qquad \psi_2^n : P_2(\varphi_2 \cup \psi_2^n)}$

Care Function

Definition (Care function $\mathfrak{C}[\![\cdot]\!]$ for T wrt $\mathfrak{P}_T^{=}[\![\cdot]\!]$)

For every set V of variables and every set ϕ of T-literals

1. $\mathfrak{C}[\![V]\!]$ maps ϕ to a care graph $\mathbf{G} = \langle V, E \rangle$.
2. If $\phi \cup \delta_{\mathbf{G}}$ is T-satisfiable for an arrangement $\delta_{\mathbf{G}}$, then for any arrangement δ_V such that $\delta_V \supseteq \mathfrak{P}_T^{=}[\![V]\!](\phi \cup \delta_{\mathbf{G}})$, it holds that $\phi \cup \delta_V$ is also T-satisfiable.

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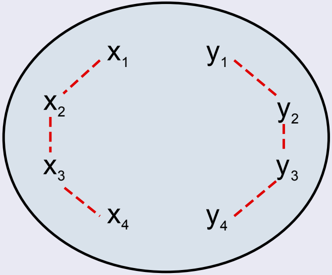
Care-Function $G = \mathfrak{C}[\![V]\!](\phi)$



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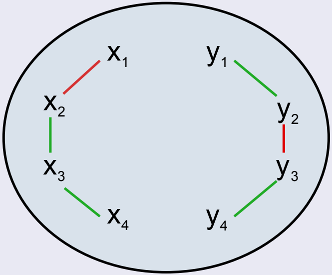
Arrangement $\delta_{\mathbf{G}}$ ($\phi \cup \delta_{\mathbf{G}}$ is SAT)



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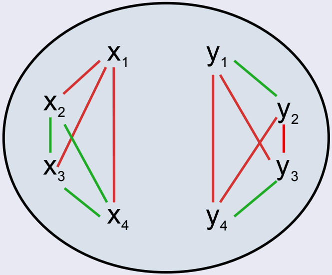
Propagation on $\phi \cup \delta_{\mathbf{G}}$



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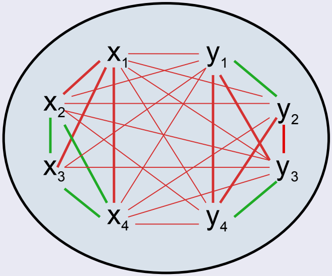

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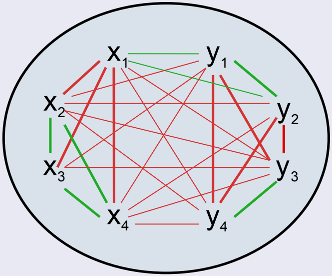

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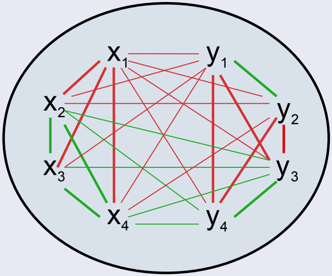

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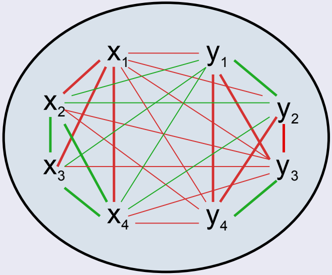

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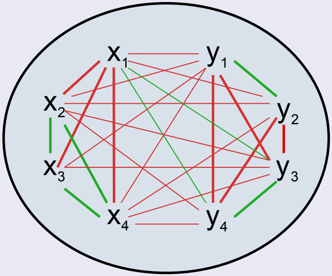

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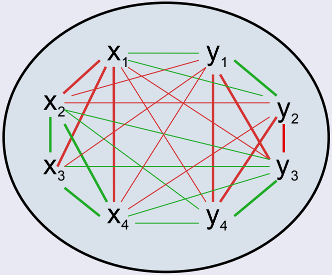

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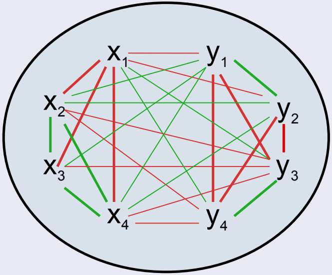


Care Function

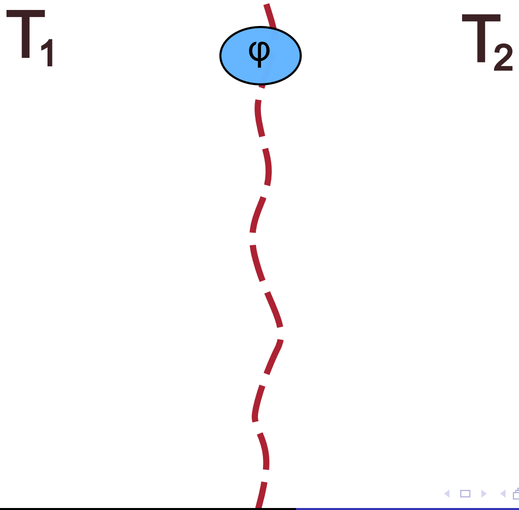
Given two care functions:

1. $\mathfrak{C}_1[\![\cdot]\!]$ for theory T_1 (with respect to $\mathfrak{P}^{=}_{T_1}[\![\cdot]\!]$)
2. $\mathfrak{C}_2[\![\cdot]\!]$ for theory T_2 (with respect to $\mathfrak{P}^{=}_{T_2}[\![\cdot]\!]$)

we can construct a care function $\mathfrak{C}[\![V]\!]$ for the combined theory $T = T_1 \oplus T_2$ (with respect to $\mathfrak{P}^{=}_{T_1}[\![\cdot]\!] \oplus \mathfrak{P}^{=}_{T_2}[\![\cdot]\!]$)as

$$\mathfrak{C}_T[\![V]\!](\phi) = \mathfrak{C}_{T_1}[\![V]\!](\phi_1) \cup \mathfrak{C}_{T_2}[\![V]\!](\phi_2) \quad .$$

Combination Method



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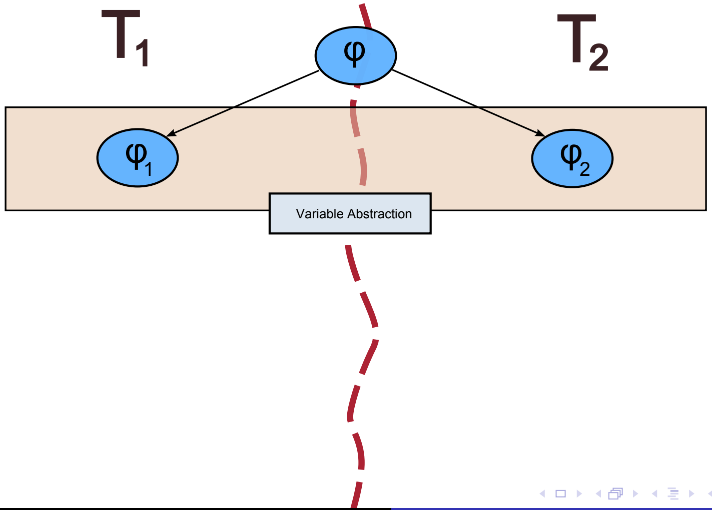

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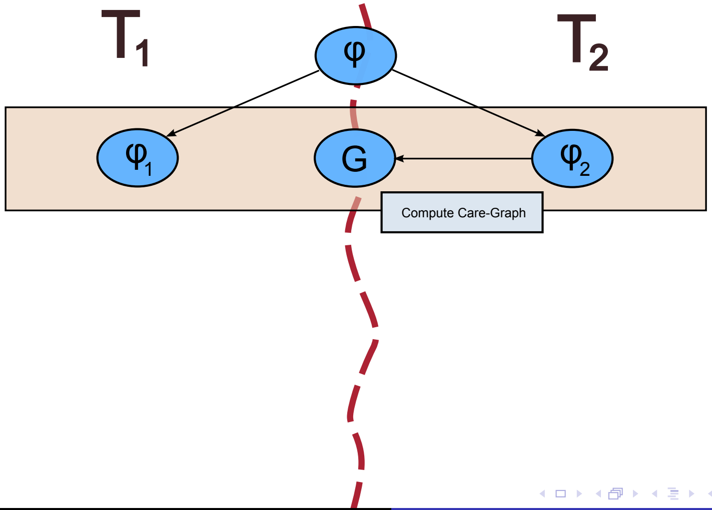

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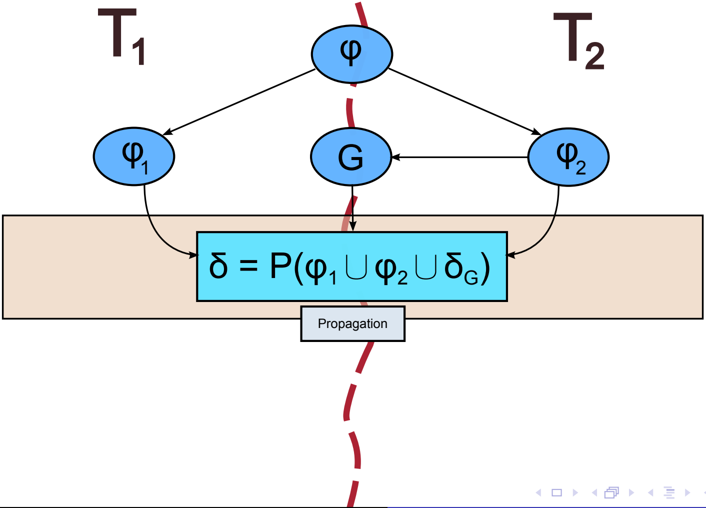

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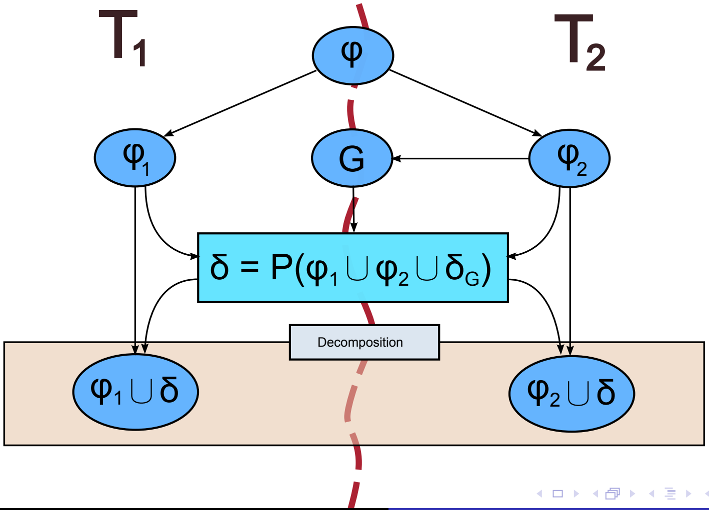
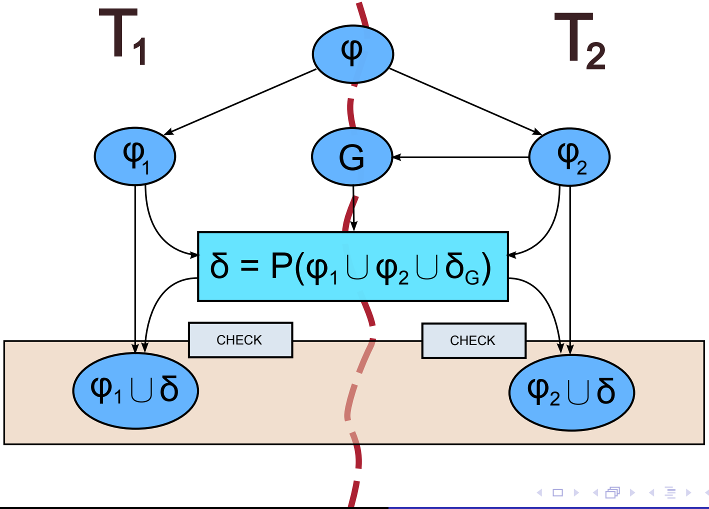


Combination Method



Theorem

1. *T_i signature-disjoint stably-infinite theory wrt S_i, $i = 1, 2$*
2. *Shared sorts are in $S_1 \cap S_2$*
3. *Care function $\mathfrak{C}_{T_i}[\![\cdot]\!]$ wrt to $\mathfrak{P}^{=}_{T_i}[\![\cdot]\!]$, $i = 1, 25$*

If the QF fragments of T_1 and T_2 are decidable, then the described procedure decides QF fragment of $T_1 \oplus T_2$.

Moreover:

- $T_1 \oplus T_2$ is stably infinite with respect to $S_1 \cup S_2$
- Equality propagator $\mathfrak{P}^{=}_{T_2}[\![\cdot]\!] \oplus \mathfrak{P}^{=}_{T_2}[\![\cdot]\!]$
- Care function $\mathfrak{C}_{T_1}[\![\cdot]\!] \oplus \mathfrak{C}_{T_2}[\![\cdot]\!]$

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Combination Method

- Equality propagation is explicit in the method
- The new method is asymmetric – only one theory computes the care graph
- Using trivial care functions and propagators the method reduces to standard Nelson-Oppen
- Easily combines with polite theory extension of Nelson-Oppen
- Care functions and propagators need to be devised for the theories

Outline

Theory of Uninterpreted Functions

- Conjunctions of literals the theory can be decided in polynomial time by congruence closure algorithms
- We make use of insights from these algorithms in defining both the equality propagator and the care function
- For simplicity, we assume that the formulas contain no predicate symbols, i.e. literals are

$$x = y, \;\; x \neq y, \;\; x = f(y_1, \ldots y_n) \;\; .$$

Equality Propagator

- Let ϕ be a set of literals, V a set of variables, and let $\sim_c$ be the smallest congruence relation over terms in ϕ containing $\{\, (x, t) \mid x = t \in \phi \,\}$.
- We define a dis-equality relation $\neq_c$ as the smallest relation satisfying

$$x \neq y \in \phi \Longrightarrow x \neq_c y \quad ,$$
$$x \sim_c x' \wedge y \sim_c y' \wedge x' \neq_c y' \Longrightarrow x \neq_c y \quad .$$

- We define the equality propagator as

$$\begin{aligned}\mathfrak{P}^{=}_{\mathtt{euf}}[\![V]\!](\phi) = & \{\, x = y \mid x, y \in V \wedge x \sim_c y \,\} \cup \\ & \{\, x \neq y \mid x, y \in V \wedge x \neq_c y \,\} \quad .\end{aligned}$$

Equality Propagator

- Let ϕ be a set of literals, V a set of variables, and let $\sim_c$ be the smallest congruence relation over terms in ϕ containing

Example

Given the set of literals

$$\phi = \{\ x = z,\ y = f(a),\ z \neq f(a)\ \}\ ,$$

the equality propagator $\mathfrak{P}^{=}_{\texttt{euf}}[\![\cdot]\!]$ would return

$$\mathfrak{P}^{=}_{\texttt{euf}}[\![x, y]\!](\phi) = \{\ x = x,\ y = y,\ x \neq y,\ y \neq x\ \}\ .$$

$$\mathfrak{P}^{=}_{\texttt{euf}}[\![V]\!](\phi) = \{\ x = y \mid x, y \in V \wedge x \sim_c y\ \} \cup \{\ x \neq y \mid x, y \in V \wedge x \not\sim_c y\ \}\ .$$

Care Function

- Let V be a set of variables and let ϕ be a set of literals
- Assume that ϕ only contains function symbols from

$$F = \{f_1, f_2, \ldots, f_n\} \ .$$

- Using the congruence relation $\sim_c$ from before, for each $f \in F$ let

$$E_f^i = \{ \ (x_i, y_i) \in V \times V \ \mid \ f(\ldots, x_i, \ldots) \nsim_c f(\ldots, y_i, \ldots) \ \} \ ,$$

  and take the care-graph edges to be $E = \bigcup_{f \in F} \bigcup_{i=1}^{arr(f)} E_f^i$.
- Define the care function as $\mathfrak{C}_{\mathtt{euf}}[\![\cdot]\!]$

$$\mathfrak{C}_{\mathtt{euf}}[\![V]\!](\phi) = \mathbf{G} = \langle V, E \rangle \ .$$

Care Function

- Let V be a set of variables and let ϕ be a set of literals
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Example

Given the set of literals

$$\phi = \{\ x_f = f(x),\ y_f = f(y)\ \}$$

with $V = \{x, x_f, y, y_f\}$, the care function would return

$$\mathfrak{C}_{\texttt{euf}}[\![V]\!](\phi) = \mathbf{G} = \langle V, \{(x, y)\}\rangle\ .$$

- Us
  $f \in$

  E_f^i

  anc
- Define the care function as $\mathfrak{C}_{\texttt{euf}}[\![\cdot]\!]$

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- Since the theory of uninterpreted functions is parametrized by the theories that define the function domains (it's using them) it makes sense for it to be in charge of the combination.
- We also have a care-function for the theory of arrays.
- Similar to uninterpreted functions – arrays are can be seen as functions with a twist

Outline

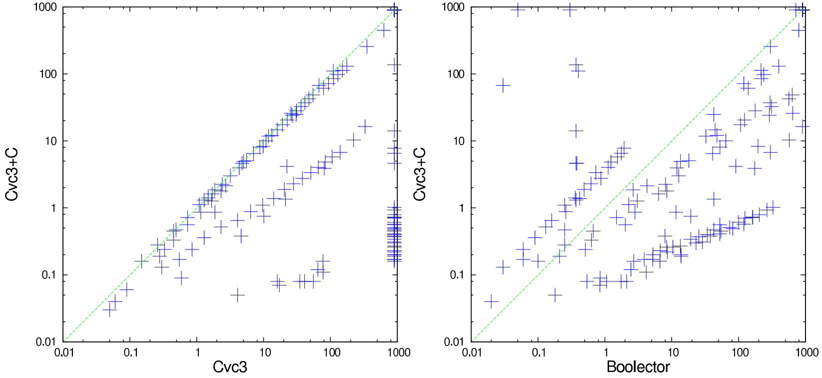
Setup

- We implemented the modified method in Cvc3 and we denote this with Cvc3+C
- The benchmarks are in the QF_AUFBV (arrays+bit-vectors)
- Benchmarks are SMT-LIB problems with more than one array variable
- Compared to Boolector, Yices, MathSAT, Z3, and Cvc3 (includes all solvers from 2009 SMT-COMP competition).

Experimental Results

	Boolector	Yices	MathSAT	Z3	Cvc3	Cvc3+C
`crafted` (40)	2100 (40)	6253 (34)	468 (30)	112 (40)	388 (9)	**14** (**40**)
`matrix` (11)	1208 (10)	683 (6)	474 (4)	927 (11)	831 (11)	**45** (**11**)
`unconstr` (10)	**3** (**10**)	(0)	706 (3)	54 (2)	185 (5)	340 (8)
`copy` (19)	11 (19)	**1** (**19**)	1103 (19)	4 (19)	432 (17)	44 (19)
`sort` (6)	691 (6)	557 (4)	82 (4)	248 (3)	**44** (**6**)	**44** (**6**)
`delete` (29)	**3407** (**18**)	1170 (10)	2626 (14)	1504 (10)	1766 (17)	1302 (17)
`member` (24)	2807 (24)	185 (24)	217 (24)	**112** (**24**)	355 (24)	320 (24)
	10229 (**127**)	8852 (97)	5678 (98)	2965 (109)	4004 (89)	2112 (125)

Experimental Results



Conclusion

- Reformulation of the classic Nelson-Oppen method for combining theories.
- Reduces the complexity of finding a common arrangement over the interface variables.
- We devised care functions for the theories of uninterpreted functions and arrays.
- The new method is asymmetric.
- Orthogonal to previous research on combinations of theories.
- Robust performance increase over the standard Nelson-Oppen implementation.

Thank you!